Finally, we point out that the application of these results is not limited to unsteady three-dimensional airfoil theory as governed by (1) subject to the approximation (2). The essential feature is the presence of the singular wake integral (together with the absence of an exact solution). The two-dimensional unsteady, as well as the steady jet-flapped, hydrofoil near a free surface represent suitable cases.

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An Analytical Approach to Hypervelocity Impact

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Nomenclature

C = sound velocity behind the shock wave given in Table III of Ref. 2
e = internal energy per unit mass
k = thermal conductivity
P = pressure
S = constant defined in Table III of Ref. 2
T = temperature
u = particle velocity
x = distance traveled by the projectile during penetration
μ = dynamic viscosity
ρ = mass density of the material

Introduction

In a previous paper by Yuan and Scully,[1] a simplified theory of penetration in hypervelocity impact was presented. This theory is based on the assumption that, during a very short interval of time, the energy released by the impact is sufficient to melt or to vaporize both the projectile and the small volume of target involved. In this regime of fluid impact, a transmitted shock is known to propagate into the target and a reflected shock propagates into the projectile. The motion of the fluid which represents the shock front is assumed to be governed by the basic equations of one-dimensional flow of a viscous compressible fluid. A solution in closed form was obtained from these equations which gives the penetration parameter as a function of the impact velocity and the sonic velocity of the material at standard conditions.

Presented as Preprint 63-192 at the AIAA Summer Meeting, Los Angeles, Calif., June 17–20, 1963; revision received May 27, 1964. This work was sponsored by Space and Information Systems Division, North American Aviation, Inc.

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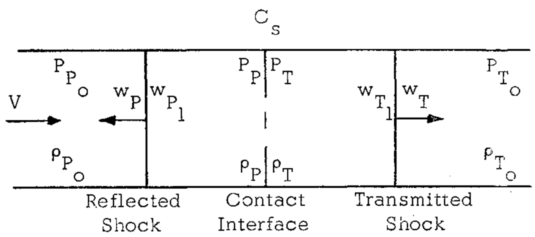

Fig. 1 Shock wave of the impact model.

Although the results of this penetration and impact velocity relation agree quite well with available experimental data for alike projectile and target materials, the equation of state for a perfect gas was used in the analysis.

In the present paper, a more realistic equation of state is introduced to replace the equation of state for a perfect gas in the basic equations for a viscous compressible fluid. This equation of state was determined through shock-wave velocity and free-surface velocity measurements. These measurements were obtained from experiments of a plane-wave explosive system in which a thin plate, moving at several kilometers per second, was used to produce strong shock waves in a stationary target plate.[2] A theoretical solution is obtained from the one-dimensional equations of a viscous compressible fluid. The results give a penetration parameter as a function of an impact velocity parameter. The constants in the theoretical expression are determined from the experimental data for identical projectile and target materials.

Based on the experimental data for three different materials, the present expression for the penetration-impact velocity relation can be simplified after dropping small terms. The result is identical to the expression for the penetration-impact velocity relation given in the previous paper.[1] Hence, it is evident that the results obtained in Ref. 1 by using the equation of state for a perfect gas are believed to be justified.

Hypervelocity Impact Model

When the stresses of the target material, caused by impact, are much greater than the yield stress, it is believed that both the projectile and target materials will eventually become fluid. As the velocity of impact approaches the same order of magnitude as the velocity of dilatational waves in the target material, shock waves are generated in the target. A conventional one-dimensional shock-wave propagation is shown in Fig. 1 to illustrate the type of impact under consideration.[3, 4]

When the projectile strikes the target at velocity V, a transmitted shock propagates into the target at velocity w_T and a reflected shock propagates into the projectile at velocity w_p. Since there is no penetration mixing of the projectile and target materials, the contact interface is established in the ideal mechanical impact model. The contact surface C_s is then separating projectile and target materials and is traveling in the direction of the transmitted shock at the particle velocity.

If the motion of the particles behind the shock front is considered steady and the shear stress is neglected, then the relation between the particle velocity behind the shock front u and the impact velocity V for alike projectile and target materials is given by[3]

$$u = V/2 \tag{1}$$

Fundamental Equations

When both the projectile and target are assumed to be in a fluid state under the high-speed impact process, the motion of the fluid which represents the shock front is governed by the following equations for one-dimensional flow of a viscous compressible fluid without a body force:

$$P = \frac{C^2\rho[(\rho/\rho_0) - 1]}{\{(\rho/\rho_0) - S[(\rho/\rho_0) - 1]\}^2} \tag{2}$$

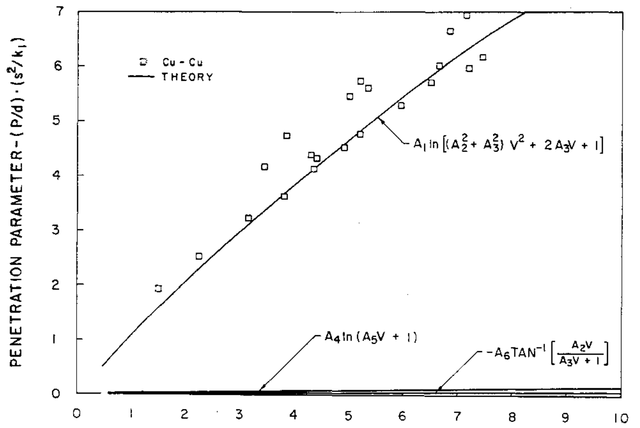

Fig. 2 Impact velocity, $V \times 10^{-3}$, fps.

$$(d/dx)(\rho u) = 0 \tag{3}$$

$$\rho u \frac{du}{dx} + \frac{dP}{dx} - \frac{4}{3}\frac{d}{dx}\left(\mu \frac{du}{dx}\right) = 0 \tag{4}$$

$$\rho u \frac{de}{dx} + P\frac{du}{dx} - \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 - \frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \tag{5}$$

Equation (2) is the Hugoniot equation of state for metallic elements from shock-wave measurements to two megabars.[2]

The integration of Eq. (3) gives the following results:

$$\rho u = \text{const} = C_1 \tag{6}$$

The combination of Eqs. (2, 4, and 6) reduces to a single equation in v in the form

$$\left[-\frac{C_1^2}{\rho_0} + C^2\rho_0\frac{(1+Sv)}{(1-Sv)^3}\right]\frac{dv}{dx} + \frac{4}{3}\frac{C_1}{\rho_0}\frac{d}{dx}\left(\mu\frac{dv}{dx}\right) = 0 \tag{7}$$

where

$$v = 1 - (\rho_0 u/C_1)$$

According to the shock-wave relation [Eq. (1)] the fluid velocity u can be expressed in terms of the velocity of impact. After lengthy integrations, Eq. (7) yields the following solution:

$$\frac{x}{d} = \frac{k_1}{S^2}\Bigg\{A_1 \ln[(A_2^2 + A_3^2)V^2 + 2A_3V + 1] + A_4\ln[A_5V + 1] + A_6\tan^{-1}\left[\frac{A_2V}{A_3V + 1}\right]\Bigg\} \tag{8}$$

where $x = p$ is the length of the penetration, d is the linear cross-sectional length of the projectile, k_1 and A's are con-

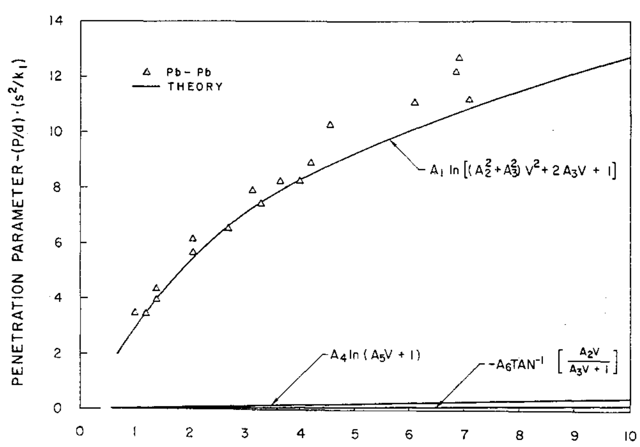

Fig. 3 Impact velocity, $V \times 10^{-3}$, fps.

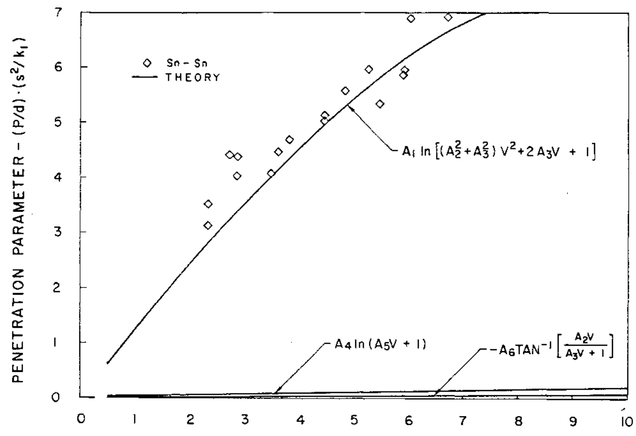

Fig. 4 Impact velocity, $V \times 10^{-3}$, fps.

stants to be determined experimentally, and S is a material constant.[2]

Correlation of Theory with Experiments

The six constants A_1, A_2, ... A_6 in Eq. (8) are very complicated in nature, and they cannot be determined directly until the physical constants of the materials at this extreme condition are known. A logical approach to the problem is to evaluate the constants A's from available experimental data of various projectile and target materials.

The experimental data on hypervelocity impact of identical projectile and target materials are rather limited. This limitation is further restricted by the materials used in the determination of the Hugoniot equation of state.[2] From an intensive search, the data that have been found useful for the present purpose are 1) the penetration of copper projectile into copper targets[5,6,8]; 2) the penetration of lead projectile into lead targets[5,6,8]; and 3) the penetration of tin projectile into tin targets.[7,8] These data are plotted in Figs. 2–4, respectively, with penetration parameter as the ordinate and the impact velocity parameter as the abscissa. With an iteration technique, the six constants are determined from the forementioned data for each of the three different materials. The calculations were carried out with the CDC 1604 digital computer. The numerical values of the A's are given in Table 1.

It is interesting to note that the preceding values of A's in all of the three cases indicate that the second and third terms on the right-hand side of Eq. (8) are much smaller in comparison with the first term. Furthermore, the term $2A_3V$ is much smaller than $(A_2^2 + A_3^2)V^2$. A comparison of the order of magnitude of these terms is shown in Figs. 2–4 for copper, lead, and tin, respectively. After neglecting all the small terms, Eq. (8) can be rewritten in the form

$$p/d = k_1 \ln[1 + A_2^2V^2] \tag{9}$$

Equation (9) is identical to Eq. (27) of Ref. 1 with $A_2^2 = k_3$.

The evaluation of k_1 and k_3 has been discussed in detail in Ref. 1. The final penetration formula can be expressed as follows:

$$\frac{p}{d} = 0.58 \ln\left[1 + \frac{(E/\rho)\times 10^{-4}}{[1.0 + 1.398 \times 10^{-3}(E/\rho)^{1/2}]^2}\cdot\frac{V^2}{(E/\rho)}\right] \tag{10}$$

where $(E/\rho)^{1/2}$ is the sonic velocity of the materials in solid

Table 1 Numerical constants in Eq. (8)

	$A_1 = S^2$	$A_2 \times 10^4$	$A_3 \times 10^{10}$	$A_4 \times 10^7$	$A_5 \times 10^4$	$A_6 \times 10^7$
Copper	2.24	5.54	− 1.8	7.56	7.62	− 5.29
Lead	2.30	15.8	13.5	1.19	0.37	− 11.4
Tin	2.18	6.68	− 7.2	13.2	1.99	− 0.26

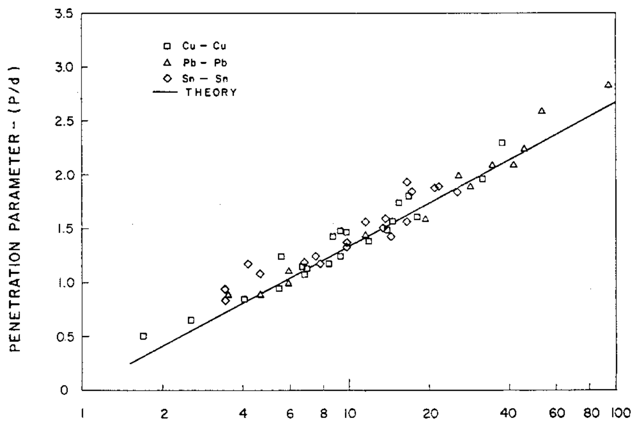

Fig. 5 Impact velocity parameter $(1 + A_2{}^2V^2)$.

state at standard conditions. For a given impact velocity and the corresponding sonic velocity of the material at standard conditions, the penetration parameter can be readily determined without the aid of any specific experimental constant.

Figure 5 summarizes all the three sets of data on hypervelocity impact on projectile and target of the same materials. These data include copper into copper, lead into lead, and tin into tin. On the graph, the ordinate is p/d, and the abscissa is the value of the logarithm of $[1 + A_2{}^2V^2]$ where

$$A_2{}^2\left(\frac{E}{\rho}\right) = \frac{(E/\rho) \times 10^{-4}}{[1.0 + 1.398 \times 10^{-3}\,(E/\rho)^{1/2}]^2} \qquad (11)$$

The theoretical expression given in Eq. (10) is represented by a single straight line on the graph. The agreement between this single theoretical curve and available experimental data is surprisingly good.

Conclusions

A more realistic equation of state is introduced to replace the equation of state for a perfect gas in the basic equations for a viscous compressible fluid. The result of the present investigation shows that the use of the equation of state for a perfect gas in the hypervelocity impact analysis is justified.

Future research on the refinements of the present theory is being planned, including the unsteady case, the calculation of the crater volume, and the case of the projectile and target being of different materials.

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Effects of Interface Combustion and Mixing on Shock-Tunnel Conditions

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Introduction

FLOW conditions in the test section of a shock tunnel are usually determined from measured test-section pitot pressures and reservoir pressures and from a reservoir enthalpy calculated from the measured shock speed. In fact, one of the advantages of the shock tunnel is that the reservoir conditions (at least immediately after shock reflection) can be calculated with confidence from the measured shock speed. The flow from the reservoir to the test section is usually assumed to be one-dimensional, isentropic, and in chemical equilibrium. Test-section conditions so computed may be in error because one or more of these assumptions are violated or because of reservoir nonuniformities (e.g., test gas dilution with driver gases) or reservoir losses (e.g., radiation), both of which become more important as reservoir enthalpy levels are increased. In order to assess the accuracy and suitability of the usual method of determining shock-tunnel flow conditions, a program has been undertaken at the Douglas Aerophysics Laboratory (DAL) to measure directly the test-section airflow velocity in a shock tunnel over a wide range of reservoir enthalpies. With this additional measurement, all test-section properties can be determined without resort to any of the previous assumptions.

Experimental Equipment and Interpretation

The experiments were performed in the DAL Hypervelocity Impulse Tunnel (HIT).[1] The driver tube has a 6-in. bore and is 34 ft long, and the driven tube has a 5-in. bore and is 30.67 ft long. The nozzle axis is rotated 90° from the shock-tube axis, they intersect 1 in. upstream of the driven tube end wall. For the series of experiments reported herein, the nozzle throat diameter was 0.840 in. and the time of uniform reservoir pressure was at least 13 msec.

The test-section airflow velocity was measured directly using a technique similar to that described in Refs. 2 and 3. A disturbance in the form of a cylindrical blast wave was generated by the discharge of electrical energy between two electrodes spaced 3 in. apart in the test section. This disturbance was convected downstream with the flow velocity, and its position was recorded a known time later on a schlieren photograph. The accuracy of this velocity measuring technique is primarily limited by the accuracy with which the blast wave displacement can be measured, which was about ±2%.

On some runs, the stagnation-point heat-transfer rate to a 1.25-in.-diam cylinder mounted transverse to the airflow was measured. Thin-film resistance-thermometer outputs were converted into heat-transfer rates by analog circuits.[4]

Test-section velocities also were calculated from the reservoir enthalpy. Conditions immediately after shock reflection (designated 5) were calculated from the measured shock speed extrapolated to the end of the driven tube. When reservoir conditions (designated R) were not the same as conditions after shock reflection (e.g., when tailored-interface conditions[5] were not achieved as is the case illustrated in Fig. 1), the process between 5 and R was assumed to be isentropic, and the reservoir enthalpy h_R was calculated from the measured reservoir pressure record using

$$h_R = h_5(p_R/p_5)^{(\gamma-1)/\gamma} \qquad (1)$$

Received May 25, 1964.

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